NCERT Solutions Class 12 Maths Chapter 2
Exercise 2.1

Question 1:

Find the principal value of $\sin^{-1}\left(-\frac{1}{2}\right)$.

Solution:

Let, $\sin^{-1}\left(-\frac{1}{2}\right)=y$

Hence,

$$\sin y=\left(-\frac{1}{2}\right)$$
$$=-\sin\left(\frac{\pi}{6}\right)$$
$$=\sin\left(-\frac{\pi}{6}\right)$$

Range of the principal value of $\sin^{-1}(x)$ is $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

Thus, principal value of $\sin^{-1}\left(-\frac{1}{2}\right)=\left(-\frac{\pi}{6}\right)$.

Question 2:

Find the principal value of $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$.

Solution:

Let, $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)=y$

Hence,

$$\cos y=\left(\frac{\sqrt{3}}{2}\right)$$
$$=\cos\frac{\pi}{6}$$

Range of the principal value of $\cos^{-1}(x)$ is $(0,\pi)$.

Thus, principal value of $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)=\left(\frac{\pi}{6}\right)$

Question 3:

Find the principal value of $\text{cosec}^{-1}(2)$.

Solution:

Let, $\text{cosec}^{-1}(2) = y$

Hence,

$$\text{cosec} y = 2$$

$$= \text{cosec}\left(\frac{\pi}{6}\right)$$

Range of the principal value of $\text{cosec}^{-1}(x) = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

Thus, principal value of $\text{cosec}^{-1}(2) = \left(\frac{\pi}{6}\right)$.

Question 4:

Find the principal value of $\tan^{-1}\left(-\sqrt{3}\right)$

Solution:

Let, $\tan^{-1}\left(-\sqrt{3}\right) = y$

Hence,

$$\tan y = -\sqrt{3}$$

$$= -\tan\left(\frac{\pi}{3}\right)$$

$$= \tan\left(-\frac{\pi}{3}\right)$$

Range of the principal value of $\tan^{-1}(x) = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Thus, principal value of $\tan^{-1}\left(-\sqrt{3}\right) = \left(-\frac{\pi}{3}\right)$.

Question 5:

Find the principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$

Solution:

Let, $\cos^{-1}\left(-\frac{1}{2}\right)=y$

Hence,

$$\cos y=-\frac{1}{2}$$
$$=-\cos\left(\frac{\pi}{3}\right)$$
$$=\cos\left(\pi-\frac{\pi}{3}\right)$$
$$=\cos\left(\frac{2\pi}{3}\right)$$

Range of the principal value of $\cos^{-1}(x)=[0,\pi]$

Thus, principal value of $\cos^{-1}\left(-\frac{1}{2}\right)=\left(\frac{2\pi}{3}\right)$.

Question 6:

Find the principal value of $\tan^{-1}(-1)$

Solution:

Let, $\tan^{-1}(-1)=y$

Hence,

$$\tan y=-1$$
$$=-\tan\left(\frac{\pi}{4}\right)$$
$$=\tan\left(-\frac{\pi}{4}\right)$$

Range of the principal value of $\tan^{-1}(x)=\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

Thus, principal value of $\tan^{-1}(-1)=\left(-\frac{\pi}{4}\right)$.

Question 7:

Find the principal value of $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$

Solution:

Let, $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)=y$

Hence,

$$\sec y=\frac{2}{\sqrt{3}}$$

$$=\sec\left(\frac{\pi}{6}\right)$$

Range of the principal value of $\sec^{-1}(x)=[0,\pi]-\left\{\frac{\pi}{2}\right\}$

Thus, principal value of $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)=\left(\frac{\pi}{6}\right)$.

Question 8:

Find the principal value of $\cot^{-1}\left(\sqrt{3}\right)$

Solution:

Let, $\cot^{-1}\left(\sqrt{3}\right)=y$

Hence,

$$\cot y=\sqrt{3}$$

$$=\cot\left(\frac{\pi}{6}\right)$$

Range of the principal value of $\cot^{-1}(x)=(0,\pi)$

Thus, principal value of $\cot^{-1}\left(\sqrt{3}\right)=\left(\frac{\pi}{6}\right)$.

Question 9:

Find the principal value of $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

Solution:

Let, $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)=y$

Hence,

$$\cos y = -\frac{1}{\sqrt{2}}$$
$$= -\cos\left(\frac{\pi}{4}\right)$$
$$= \cos\left(-\frac{\pi}{4}\right)$$
$$= \cos\left(\pi - \frac{\pi}{4}\right)$$
$$= \cos\left(\frac{3\pi}{4}\right)$$

Range of the principal value of $\cos^{-1}(x) = [0, \pi]$

Thus, principal value of $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) = \left(\frac{3\pi}{4}\right)$.

Question 10:

Find the principal value of $\text{cosec}^{-1}\left(-\sqrt{2}\right)$

Solution:

Let, $\text{cosec}^{-1}\left(-\sqrt{2}\right) = y$

Hence,

$$\text{cosec}\, y = -\sqrt{2}$$
$$= -\text{cosec}\left(\frac{\pi}{4}\right)$$
$$= \text{cosec}\left(-\frac{\pi}{4}\right)$$

Range of the principal value of $\text{cosec}^{-1}(x) = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

Thus, principal value of $\text{cosec}^{-1}\left(-\sqrt{2}\right) = \left(-\frac{\pi}{4}\right)$.

Question 11:

Find the value of $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$.

Solution:

Let, $\tan^{-1}(1) = x$

Hence,

$$\tan x = 1$$
$$= \tan\left(\frac{\pi}{4}\right)$$

Therefore,

$$\tan^{-1}(1) = \left(\frac{\pi}{4}\right)$$

Now, let $\cos^{-1}\left(-\frac{1}{2}\right) = y$

Hence,

$$\cos y = -\frac{1}{2}$$
$$= -\cos\left(\frac{\pi}{3}\right)$$
$$= \cos\left(\pi - \frac{\pi}{3}\right)$$
$$= \cos\left(\frac{2\pi}{3}\right)$$

Therefore,

$$\cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$$

Again, let $\sin^{-1}\left(-\frac{1}{2}\right) = z$

Hence,

$$\sin z = -\frac{1}{2}$$
$$= -\sin\left(\frac{\pi}{6}\right)$$
$$= \sin\left(-\frac{\pi}{6}\right)$$

Therefore,

$$\sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$$

Thus,

$$\tan^{-1}(1)+\cos^{-1}\left(-\frac{1}{2}\right)+\sin^{-1}\left(-\frac{1}{2}\right)=\frac{\pi}{4}+\frac{2\pi}{3}-\frac{\pi}{6}$$
$$=\frac{3\pi+8\pi-2\pi}{12}$$
$$=\frac{9\pi}{12}$$
$$=\frac{3\pi}{4}$$

Question 12:

Find the value of $\cos^{-1}\left(\frac{1}{2}\right)+2\sin^{-1}\left(\frac{1}{2}\right)$

Solution:

Let, $\tan^{-1}(1)=x$

Hence,

$$\cos x=\frac{1}{2}$$
$$=\cos\left(\frac{\pi}{3}\right)$$

Therefore,

$$\cos^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{3}$$

Let, $\sin^{-1}\left(\frac{1}{2}\right)=y$

Hence,

$$\sin y=\frac{1}{2}$$
$$=\sin\left(\frac{\pi}{6}\right)$$

Therefore,

$$\sin^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{6}$$

Thus

$$\cos^{-1}\left(\frac{1}{2}\right)+2\sin^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{3}+2\left(\frac{\pi}{6}\right)$$
$$=\frac{2\pi}{3}$$

Question 13:

Find the value of $\sin^{-1}x=y$, then

(A) $0\le y\le\pi$ (B) $-\frac{\pi}{2}\le y\le\frac{\pi}{2}$

(C) $0\le y\le\pi$ (D) $-\frac{\pi}{2}<y<\frac{\pi}{2}$

Solution:

It is given that $\sin^{-1}x=y$

Range of the principal value of $\sin^{-1}x=\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

Thus, $-\frac{\pi}{2}\le y\le\frac{\pi}{2}$

The answer is B.

Question 14:

Find the value of $\tan^{-1}\sqrt{3}-\sec^{-1}(-2)$ is equal to

(A) 0 (B) $-\frac{\pi}{3}$

(C) $\frac{\pi}{3}$ (D) $\frac{2\pi}{3}$

Solution:

Let $\tan^{-1}\left(\sqrt{3}\right)=x$

Hence,

$$\tan x=\sqrt{3}$$
$$=\tan\left(\frac{\pi}{3}\right)$$

Range of the principal value of $\tan^{-1} x = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Therefore, $\tan^{-1}\left(\sqrt{3}\right) = \left(\frac{\pi}{3}\right)$

Let $\sec^{-1}(-2) = y$

Hence,

$$\sec y = (-2)$$
$$= -\sec\left(\frac{\pi}{3}\right)$$
$$= \sec\left(-\frac{\pi}{3}\right)$$
$$= \sec\left(\pi - \frac{\pi}{3}\right)$$
$$= \sec\left(\frac{2\pi}{3}\right)$$

Range of the principal value of $\sec^{-1}x = [0, \pi] - \left\{\frac{\pi}{2}\right\}$

Therefore, $\sec^{-1}(-2) = \frac{2\pi}{3}$

Thus,

$$\tan^{-1}\sqrt{3} - \sec^{-1}(-2) = \frac{\pi}{3} - \frac{2\pi}{3}$$
$$= -\frac{\pi}{3}$$

The answer is B.